Rocked thermal ratchets: The high-frequency limit

J. Plata
Departamento de Física Fundamental y Experimental, Universidad de La Laguna, La Laguna E38204, Tenerife, Spain
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The generation of probability currents in periodically driven asymmetric periodic potentials is studied in the high-frequency regime. Approximate values for the mean current are obtained by using a general analytic treatment of the Fokker-Planck equation and an approximation based on the existence of two widely different relevant time scales in the problem. The results allow us to understand how the ratchet effect tends to disappear as the frequency grows. The same treatment is applied to tilted potentials. [S1063-651X(98)11005-X]

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I. INTRODUCTION

The generation of probability currents in fluctuating nonequilibrium systems has recently attracted considerable attention. In particular, the analysis of the fundamental basis and the possible applications of fluctuation-induced transport in driven asymmetric periodic potentials (ratchets) has been the subject of intense work. Much of this research has been carried out using a variety of models set up by changing the driving terms and the statistics of the noise in the basic system in which the phenomenon was initially studied [1]. In this way, different aspects of the effect have been considered, and interesting results, such as a peak structure in the dependence of the current on the noise intensity, and current reversals obtained by varying the magnitude of noise or the amplitude of the driving term [2–4], have been found. There are still some open questions, and in this respect emphasis has been put on the need of analytical explanations for some of the findings, and on a more detailed understanding of the fast-forcing limit. Apart from their intrinsic fundamental interest, some of these models can be relevant to analyze processes in biology, chemistry, and physics.

Here we have focused on a *rocked thermal ratchet*, that is, a ratchet with Gaussian white noise and a zero-mean periodic driving force. This simple and generic model, which has been studied analytically in the adiabatic regime and numerically for intermediate frequencies [4], is especially appropriate to analyze in a straightforward way the cooperative effect on the appearance of a net current, of time correlations, broken spatial symmetry, and noise. Our objective has been to make analytical and numerical studies of the high-frequency limit, and identify the range of values for the amplitude and frequency of the driving term, in which the ratchet effect persists.

The outline of the paper is as follows. In Sec. II, following the method presented in Ref. [5], the spatial periodicity of the potential is used to rewrite the Fokker-Planck equation as a recurrence relation, and to obtain the mean current in terms of two coefficients of the Fourier series for the probability density. The high-frequency limit is considered in Sec. III: we analyze which terms in the recurrence relation are relevant to the generation of probability currents, and discuss their dependence on the parameters of the driving term. In Sec. IV the method is applied to implement a partial control of the dynamics by changing the shape of the effective potentials. The subject of Sec. V is the discussion, in the same framework, of ratchet effect in tilted systems. Finally, in Sec. VI some conclusions are summarized.

II. GENERAL THEORY

We consider an overdamped particle moving in an asymmetric periodic potential driven by a time-periodic term, and under the action of thermal noise. Specifically we have studied the system defined by the Langevin equation

$$\dot{x}=-\partial_x\{V(x)-xF(t)\}+\xi(t), \tag{1}$$

where the ratchet potential is given by $V(x)=-k^{-1}[b_1\sin(kx)+b_2\sin(2kx)]$ [4], for the driving force we have taken $F(t)=A\sin(\omega t)$, and $\xi(t)$ is Gaussian white noise, i.e.,

$$\langle\xi(t)\rangle=0,$$
$$\langle\xi(t)\xi(t')\rangle=2D\,\delta(t-t'). \tag{2}$$

The Fokker-Planck equation for the probability density $W(x,t)$ is readily obtained [6], and, in terms of the probability current $J(x,t)$, is given by

$$\frac{\partial W(x,t)}{\partial t}+\frac{\partial J(x,t)}{\partial x}=0, \tag{3}$$

the current for our system being

$$J(x,t)=\{b_1\cos(kx)+2b_2\cos(2kx)+A\ \sin(\omega t)-D\partial_x\}W(x,t). \tag{4}$$

As translational invariance is assumed, periodic boundary conditions are added, $W(x,t)=W(x+2\pi/k,t)$, and, making use of this periodicity, the probability distribution function is expanded in a Fourier series

$$W(x,t)=\sum_{n=-\infty}^{\infty}C_n(t)e^{inkx} \tag{5}$$

whose normalization to one over the spatial period gives the value $C_0=k/2\pi$.

Our aim is to obtain the mean current in the asymptotic limit, $t\to\infty$, in which a unique limiting distribution, independent of the initial conditions, and periodic in time and space, is reached. Therefore, according to the method presented in Ref. [5], we make the following transformation, which will be particularly useful in the high-frequency limit:

$$\tilde{c}_n = C_n \exp\left[-in\frac{kA}{\omega}\cos(\omega t)\right]. \tag{6}$$

With this ansatz and the Bessel expansion [7]

$$e^{iz\cos\phi} = \sum_{l=-\infty}^{\infty} i^l J_l(z) e^{il\phi}, \tag{7}$$

Eq. (3) is reduced to

$$\begin{aligned}\dot{\tilde{c}}_n = & -n^2k^2D\tilde{c}_n - ink(b_1/2)\sum_{l=-\infty}^{\infty} i^l J_l(kA/\omega) \\ & \times e^{il\omega t}[(-1)^l\tilde{c}_{n-1} + \tilde{c}_{n+1}] - inkb_2 \\ & \times \sum_{l=-\infty}^{\infty} i^l J_l(2kA/\omega)e^{il\omega t}[(-1)^l\tilde{c}_{n-2} + \tilde{c}_{n+2}],\end{aligned} \tag{8}$$

and finally, the mean current $\overline{J}$, that is, the probability current doubly averaged over the spatial and temporal periods, is given by

$$\begin{aligned}\overline{J} \equiv & \frac{k}{2\pi}\frac{\omega}{2\pi}\int_t^{t+2\pi/\omega} dt' \int_0^{2\pi/k} J(x,t')dx \\ = & b_1 \sum_{l=-\infty}^{\infty} J_l(kA/\omega)\mathrm{Re}\{i^l\langle\tilde{c}_1 e^{il\omega t}\rangle_T\} \\ & + 2b_2\sum_{l=-\infty}^{\infty} J_l(2kA/\omega)\mathrm{Re}\{i^l\langle\tilde{c}_2 e^{il\omega t}\rangle_T\}.\end{aligned} \tag{9}$$

No approximations have been made until now: Eqs. (8) and (9) have general validity. The functional form of Eq. (8) is especially appropriate to implement successive orders of approximation to the exact solution: in effect, the nondiagonal couplings are expressed as series of Bessel functions of increasing order multiplied by increasingly higher time harmonics, and, given the behavior of the Bessel functions for small arguments, $kA \ll \omega$, [7], a perturbative scheme can be set up.

The interplay of deterministic terms and fluctuations in the dynamics is clear from Eqs. (8) and (9): systems with different noise intensities have the same mean current as a function of ω when their deterministic dynamics differ in a proper scale factor.

It is also interesting to point out that, because of the double periodicity, in space and time, of the system, a double Fourier expansion can be used for the probability density in the asymptotic limit. Subsequently, the mean current, given by a compact expression of the coefficients, can be obtained by an infinite-matrix continued fractions calculation. Nevertheless, given our interest in studying the high-frequency limit, we have opted to keep explicitly the time dependence in the coefficients: in this way we can now introduce an approximation that will allow us to go further in the analytic treatment.

III. HIGH-FREQUENCY LIMIT

Our objective in this section is to identify in the recurrence relation of Eq. (8), the terms which are relevant to the generation of a net current. To this end it is useful to start considering the system in a regime defined by the relations

$$\begin{aligned} n^2k^2D \ll \omega, \quad & n=1,2, \\ nkb_i \ll \omega, \quad & i=1,2. \end{aligned} \tag{10}$$

In this case, there are two widely separate characteristic times in the system. First the time associated with the fluctuations, $1/(k^2D)$, and second, the period of the driving force, $\tau = 2\pi/\omega$. Because of the difference between both time scales, we can approximately solve Eq. (8) in two steps: first, we make an average over the driving period, keeping only the slowly varying contribution due to noise; and then we obtain the stationary limit in the averaged equation.

From the analysis of the relative contribution of the diagonal and nondiagonal terms of Eq. (8) when the conditions given by Eq. (10) are met, it is found that the time averages over one period of the driving force can be approximated as

$$\langle e^{il\omega t}\tilde{c}_n(t)\rangle_T = \frac{1}{\tau}\int_t^{t+\tau} e^{il\omega t'}\tilde{c}_n(t')dt' \cong \tilde{c}_n(t)\delta_{l,0}. \tag{11}$$

It is evident that with this approximation, equivalent to a coarse graining, it is assumed that noise has a negligible effect on the dynamics during a driving period.

In this framework the secular variation of the coefficients is readily obtained, and it follows that, in the steady state limit,

$$\begin{aligned} 0 = & -n^2k^2D\tilde{c}_n^{\infty} - ink(b_1/2)J_0(kA/\omega)(\tilde{c}_{n-1}^{\infty} + \tilde{c}_{n+1}^{\infty}) \\ & - inkb_2J_0(2kA/\omega)(\tilde{c}_{n-2}^{\infty} + \tilde{c}_{n+2}^{\infty}). \end{aligned} \tag{12}$$

It is straightforward to check that this last equation corresponds to the stationary dynamics of an overdamped particle moving in the potential $V(x) = -k^{-1}[b_1J_0(kA/\omega)\sin(kx) + b_2J_0(2kA/\omega)\sin(2kx)]$ and under the action of the Gaussian white noise defined by Eq. (2). For this effective system the stationary probability density is easily obtained [6], the net current being trivially zero as it corresponds to a system in equilibrium. Therefore, detailed balance is approximately recovered for the nonreduced system in the studied regime, and the effects of the driving term are merely a *renormalization* of the *bare* potential and the introduction in the coefficients C_n of the explicit time dependence given by Eq. (6). We must remark that the absence of a ratchet effect is implicit in the assumption made about the negligible effect of noise over the time scale of the driving force: given the values assumed for the noise intensity, time correlations are not sufficiently slow to be effective in the definition of a preferred direction in the asymmetric potential.

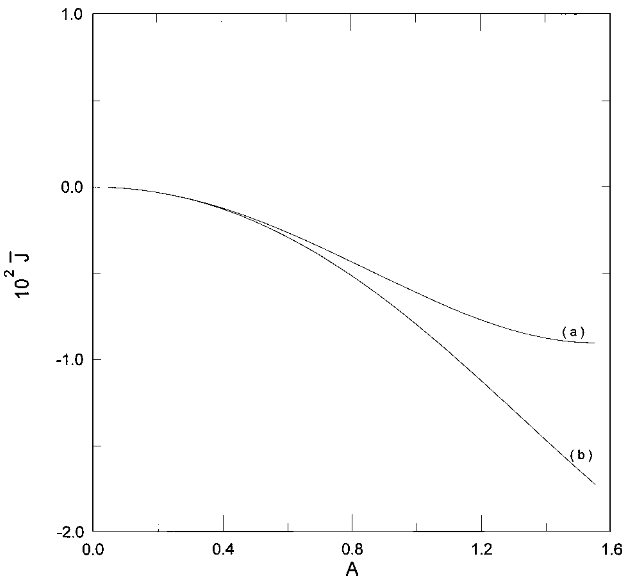

FIG. 1. Probability current vs the amplitude of the driving force A for the parameters $b_1=1$, $b_2=0.25$, and $k=2\pi$ (these potential parameters are used in all the figures); also, $\omega=10$ and $D=0.1$. (a) Exact results. (b) First-order approximation results.

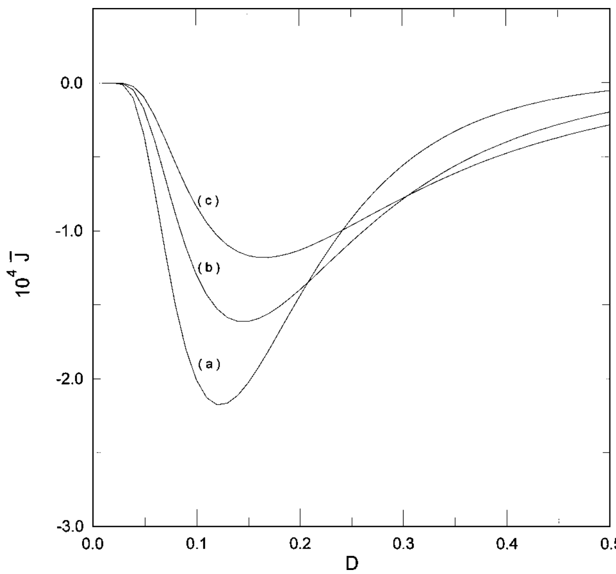

FIG. 2. Probability current vs noise strength D for $kA/\omega=0.1$, $\omega=20$ (a), $\omega=30$ (b), and $\omega=40$ (c). The results of the first-order approximation fall into the exact curves.

Information about the dynamics of the system in other regimes can be obtained by analyzing the terms neglected in the study. From the above results, which correspond to a zero-order approximation to the solution of Eq. (8), it is evident that the terms given by Bessel functions of order different from zero are responsible for the existence of net currents in any regime. Therefore, the relevance to the dynamics of at least the first-order terms is a necessary condition for the presence of a net drift speed. Given the properties of the Bessel functions $J_1(kA/\omega)$ and $J_1(2kA/\omega)$ for small arguments, $kA \ll \omega$, [7], we can conclude that, in the fast-forcing regime for a fixed value of the amplitude of the driving force, the ratchet effect tends to disappear for increasing frequencies. Note that the oscillations of the Bessel functions can lead to a nontrivial behavior of the mean velocity for intermediate frequencies when the amplitude or frequency of the driving force is varied. Obviously as the frequency decreases more terms are necessary to give a good description of the system, and explicit analytic solutions become more difficult.

A more complete estimation of the magnitude of the perturbative terms can be made analytically by using the zero-order probability density as the weight function. In this way approximate compensation of higher-order terms can explain the disappearance of net transport for certain sets of parameters. Nevertheless, a more quantitative ground is needed for a detailed analysis of the system. In order to achieve this, and taking into account that the specific dependence of the net currents on the coefficients requires high-accuracy methods in the numerical solution of the problem, we have solved the complete system and its first-order approximation version using an infinite-matrix continued fraction calculation from the framework given by Eqs. (8) and (9).

Our results illustrate the key role played by the argument kA/ω in defining the effectiveness of the time correlations. In effect, in Fig. 1, where we present the mean currents vs A for fixed values of the frequency and noise intensity, it can be seen that as kA/ω grows the driving is effectively activated in the system and there is an increase in the magnitude of the current; for small arguments the first-order approximation gives a good description of the dynamics. Eventually, as the Bessel functions $J_2(kA/\omega)$ and $J_2(2kA/\omega)$ reach significant values, the agreement between the exact and approximate results is lost. In Fig. 2, we plot the currents vs D for different frequencies and the same kA/ω. We had advanced that in the weak noise regime, even with a fixed value of this argument, the importance of the nondiagonal terms in the recurrence relations must diminish as the frequency grows. The results presented in Fig. 2 corroborate this assumption: for small values of D, when the frequency is increased there is an attenuation of the net transport. Note also the behavior found when the time associated with the fluctuations is comparable to the driving period: for increasing frequencies the maximum in the magnitude of the current and the saturation point of the ratchet effect are both shifted toward higher noise intensities.

IV. DRESSED POTENTIALS: EFFECTIVE PROBABILITY DENSITIES

In this section we want to explore the possibility of using the previous results to alter the dynamics of the system in a controlled way. It was concluded that in the zero-order approximation the effect of the driving term is a *renormalization* of the *bare* potentials. In the high-frequency limit this effect gives rise to a reduction of the barriers, and therefore to a higher effective diffusion constant. It is nevertheless outside this regime where the zero-order Bessel functions differ significantly from 1, and consequently where nontrivial changes in the shape of the potentials can be observed.

We want to show that even outside the range of validity of the zero-order approximation the method presented provides us with a certain predictive power to implement a partial control of the system by properly choosing the parameters of the driving term. In this sense, the following ideas

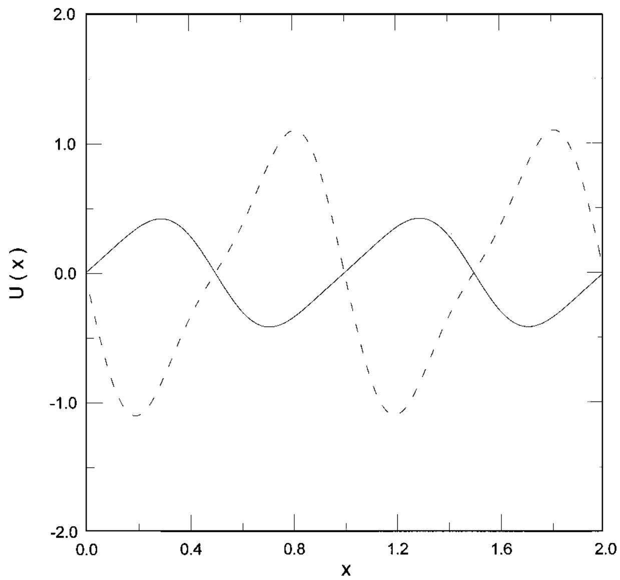

FIG. 3. Bare (dashed curve) and dressed (solid curve) potentials, for $\omega=20$, $kA/\omega=3.8317$, and $D=0.1$.

must be taken into account. First, the possibility of choosing the weight of one of the time dependent terms in Eq. (8) can always be used to optimize a particular effect. Second, the analytic knowledge of the zero-order probability density allows us to predict approximately the magnitude of any term in the recurrence relation provided the perturbative scheme is still valid. Finally, the relevance of all the time dependent terms can be globally reduced by working with sufficiently high frequencies.

The results of an example of practical use of these ideas are presented in Figs. 3 and 4. In Fig. 3, the *bare* potential is compared with the *dressed* potential for a particular choice of the parameters of the system. In Fig. 4, the values of the probability density in the zero-order approximation are compared with the exact ones. With this example we are focusing on one aspect of the dynamics that can be controlled: the system can be lead to a completely different equilibrium configuration in which detailed balance is approximately recovered, and where the positions of the maxima and minima have substantially changed. In effect, for the case studied, the probability current is very small, $\overline{J}=-3\times10^{-5}$, and the distribution function hardly oscillates around its average (zero-order) value. This dynamical behavior has been forced by taking $kA/\omega=3.8317$, which, being a zero of the first-order Bessel function and leading to a small value for $J_1(2kA/\omega)$, reduces the effect of the first-order time correlations in a significant way; additionally, the zero-order distribution function has allowed us to predict the approximate compensation of the second-order terms. Finally, we have minimized the effect of the higher-order terms by working with a high frequency, $\omega=20$.

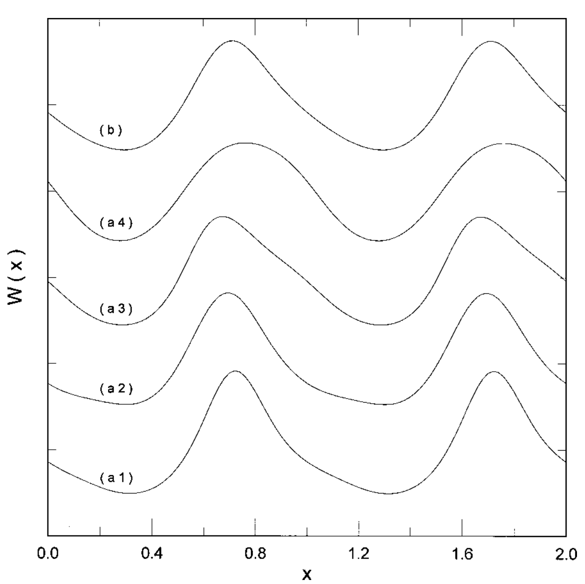

FIG. 4. Probability density for $\omega=20$, $kA/\omega=3.8317$, and $D=0.1$. Exact stationary results for $t=0$ (a1), $t=\tau/4$ (a2), $t=\tau/2$ (a3), and $t=3\tau/4$ (a4); zero-order approximation results (b). The origin in the vertical axis is arbitrary.

V. RATCHET EFFECT IN TILTED SYSTEMS

Recently, the existence of *ratchet effects* in biased asymmetric periodic potentials has been pointed out [8]. For these potentials, the combined effect of broken spatial symmetry and time correlations produces qualitative changes in the mean velocity of the system when compared with the symmetric case, and even *uphill* movement has been found for a small bias and an *appropriate* orientation of the ratchet. In order to discuss the persistence of these effects in the high-frequency limit, we have applied the above treatment to the system defined by the Langevin equation

$$\dot{x}=a+b_1\cos(kx)+2b_2\cos(2kx)+A\ \sin(\omega t)+\xi(t). \tag{13}$$

Following the method introduced in Ref. [5], we write the bias force a in the form

$$ka=r\omega+k\tilde{a}, \tag{14}$$

where r is an integer. Now making a transformation similar to the one given by Eq. (6), after minor algebra, for the coefficients of the probability density we obtain the differential recurrence relation

$$\begin{aligned}\dot{\tilde{c}}_n=&-(ink\tilde{a}+n^2k^2D)\tilde{c}_n-ink(b_1/2)\sum_{l=-\infty}^{\infty} i^{l+r}J_{l+r}(kA/\omega)\\&\times[(-1)^{l+r}e^{-il\omega t}\tilde{c}_{n-1}+e^{il\omega t}\tilde{c}_{n+1}]\\&-inkb_2\sum_{l=-\infty}^{\infty} i^l(-1)^rJ_{l+2r}(2kA/\omega)\\&\times[(-1)^le^{-il\omega t}\tilde{c}_{n-2}+e^{il\omega t}\tilde{c}_{n+2}],\end{aligned} \tag{15}$$

and, for the mean current in the tilted system,

$$\overline{J}=\frac{k}{2\pi}a+b_1\sum_{l=-\infty}^{\infty}J_{l+r}(kA/\omega)\mathrm{Re}\{i^{l+r}\langle\tilde{c}_1e^{il\omega t}\rangle_T\}$$

$$+2b_2\sum_{l=-\infty}^{\infty}J_{l+2r}(2kA/\omega)(-1)^r\mathrm{Re}\{i^l\langle\tilde{c}_2e^{il\omega t}\rangle_T\}. \tag{16}$$

Again, if the high-frequency regime defined by the conditions of Eq. (10) is considered, and, adding in this case the restriction

$$nk\tilde{a}\ll\omega,\quad n=1,2, \tag{17}$$

an approximation similar to the one already presented for unbiased potentials can be made. In this framework we obtain the following equation for the coefficients $\tilde{c}_n^{\infty}$ of the probability distribution function in the steady state limit

$$0=-(ink\tilde{a}+n^2k^2D)\tilde{c}_n^{\infty}-ink(b_1/2)J_r(kA/\omega)i^r$$
$$\times[\tilde{c}_{n-1}^{\infty}(-1)^r+\tilde{c}_{n+1}^{\infty}]$$
$$-inkb_2J_{2r}(2kA/\omega)(-1)^r(\tilde{c}_{n-2}^{\infty}+\tilde{c}_{n+2}^{\infty}), \tag{18}$$

and, being consistent with the approximation made for the coefficients, the mean current in the tilted system for $t\rightarrow\infty$, is given by

$$\overline{J}=\frac{k}{2\pi}a+b_1J_r(kA/\omega)\mathrm{Re}\{\tilde{c}_1^{\infty}i^r\}$$
$$+2b_2J_{2r}(2kA/\omega)(-1)^r\mathrm{Re}\{\tilde{c}_2^{\infty}\}. \tag{19}$$

From these equations it is clear that in this approximation the dynamics in the rocked thermal biased ratchet is reduced to a process of diffusion without time correlations in an effective biased periodic potential, the *bare* potential *dressed* by the driving term. Therefore it is evident that ratchet effect, understood as the appearance of an effective *gradient* generated by the interplay between spatial asymmetry and nonequilibrium driving, does not exist for the nonreduced system in the described regime. The stationary probability distribution function for the effective system can be obtained in terms of a double quadrature [6], and the existence of detailed balance is determined by the values of the bias a, the renormalized coefficients $b_1J_r(kA/\omega)$ and $b_2J_{2r}(2kA/\omega)$, and the noise strength D. The possible presence of net currents in this system is simply rooted in the bias term.

This analysis, which defines the validity of the zero-order approximation, also gives an analytical criterion to identify and evaluate the terms responsible for the existence of ratchet behavior in any regime. Indeed, we notice that it is necessary to keep at least first-order terms, that is, given in this case by $J_{r\pm1}(kA/\omega)$ and $J_{2r\pm1}(2kA/\omega)$, to have effective time correlations. Restricting ourselves to the particular case of small bias ($|ka|<\omega$) ($r=0$), interesting for the study of possible *uphill* movement, we see that again the behavior of Bessel functions of order different from zero for small arguments, $kA\ll\omega$ [7], reveals that, in the fast-forcing regime, for a fixed value of the driving amplitude, the ratchet effect is attenuated for growing frequencies.

One important characteristic of the dynamics in driven tilted potentials is the presence of steplike structure in the dependence of the current on the bias [5,8]. Equations (18) and (19) give insight into some features of this behavior in ratchet potentials. For each r there is a different effective potential. In the deterministic case the system is locked when $\tilde{a}$ is smaller than the potential barrier; therefore, the widths of the Shapiro steps can be obtained in terms of the coefficients $b_1J_r(kA/\omega)$ and $b_2J_{2r}(2kA/\omega)$. Obviously, as in the absence of asymmetry, the steps take place only at $r\omega$ values. The suppression of one of these *dead bands* can be done for symmetric potentials by choosing a zero point of the Bessel function; in the asymmetric case, a control of the widths of these bands can still be achieved by a proper choice of A and ω. The presence of noise can unlock the system: the discontinuous jumps disappear and the widths diminish. Analytical solutions for the stochastic case can be obtained for values of the parameters that lead to a symmetric effective potential [5]. Evidently this picture is only valid in the regime of high ω. Outside this range, the more complex deterministic dynamics and its interplay with fluctuations can give rise to nontrivial effects [8].

VI. CONCLUDING REMARKS

The method presented in this work provides a perturbative scheme in which some aspects of the dynamics of rocked thermal ratchets in the high-frequency regime can be properly understood. We have found that the effectiveness of time correlations to generate net transport in the studied systems is conditioned by two main factors. First, there is the argument kA/ω, which determines the effective weights of the time dependent terms in the recurrence relations. Second, there is the relative magnitude of the time associated with fluctuations $1/(k^2D)$ and the driving period $2\pi/\omega$: when the period decreases, higher noise intensities are needed to effectively activate the cooperative effect of driving and broken spatial symmetry. There are two main implications of these results. First, in the range of small arguments, the effective weights of the time dependent terms are given by the functions $J_1(kA/\omega)$ and $J_1(2kA/\omega)$; consequently the current in an unbiased potential tends to vanish as kA/ω diminishes. Second, in the studied regime of high frequency and weak noise, there is a decrease in the effectiveness of the time correlations that is intrinsically linked to any increase in frequency: for a fixed value of kA/ω, the net transport slows down for increasing frequencies. The same functional behavior determines the attenuation of ratchet effects in slightly tilted potentials. The partially analytic character of the method has allowed the identification, valid in any regime, of systems with different noise intensities and scaled deterministic dynamics as equivalent from the point of view of the generation of probability currents.

Our study has also revealed that, in the zero-order approximation, the role of the driving force in the dynamics is just a *renormalization* of the potential. This result, interesting in itself, can be used in this or in any other context to implement effective changes in potentials by using appropriate driving terms. Indeed, we have presented an example in which this idea has been used to control the dynamics partially, and drive the system to a different equilibrium configuration. In the same way, a control is feasible over the widths of the Shapiro steps in biased systems.

It is worthwhile to point out that in the first-order approximation the studied systems correspond to multiplicatively driven thermal ratchets. Our results for these model systems can be interesting as a first step in the study of the role of multiplicative colored noise, in particular quasiharmonic noise, in the generation of net macroscopic currents. A rich dynamics is expected outside the range we have focused on here.

Finally, we want to remark that the behavior found in our study is robust when variations of the shape of the potential are considered. In effect, each harmonic of the Fourier series for a generic periodic potential leads, in the recurrence relation for the coefficients, to contributions characterized by Bessel functions with arguments given by kA/ω multiplied by the order of the harmonic, and the analysis made in this work can be repeated with the same qualitative results.

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